Christ Church Grammar School

MATHEMATICS METHODS Year 12
Section One:
Calculator-free

Your name _____

Teacher's name_____

Time and marks available for this section
Reading time before commencing work: 2 minutes
Working time for this section: 15 minutes
Marks available: 15 marks

Materials required/recommended for this section
To be provided by the supervisor
This Question/Answer Booklet
Formula Sheet

To be provided by the candidate
Standard items: pens (blue/black preferred), pencils (including coloured), sharpener, correction fluid/tape, eraser, ruler, highlighters

Special items: nil

Important note to candidates
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6. **Show all your working clearly**. Your working should be in sufficient detail to allow your answers to be checked readily and for marks to be awarded for reasoning. Incorrect answers given without supporting reasoning cannot be allocated any marks. For any question or part question worth more than two marks, valid working or justification is required to receive full marks. If you repeat an answer to any question, ensure that you cancel the answer you do not wish to have marked.

7. It is recommended that **you do not use pencil**, except in diagrams.

Question 1 (6 marks)

(a) The function with rule $g(x)$ has derivative $g'(x) = \sin(2\pi x)$.
Given that $g(1) = \frac{1}{\pi}$, find $g(x)$. (3 marks)

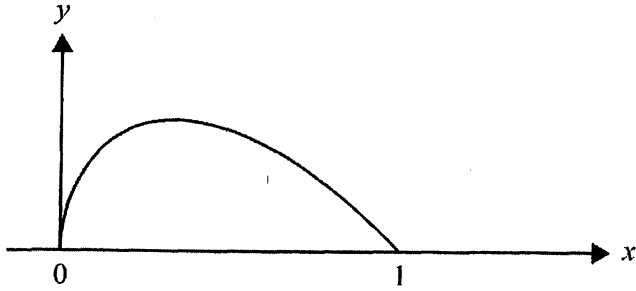(b) If $\int_1^4 f(x)\,dx = 6$, then find the value of $\int_1^4 (5 - 2f(x))\,dx$. (3 marks)

See next page

Question 2 **(4 marks)**

The graph of $f(x) = \sqrt{x}(1 - x)$ for $0 \leq x \leq 1$ is shown below.

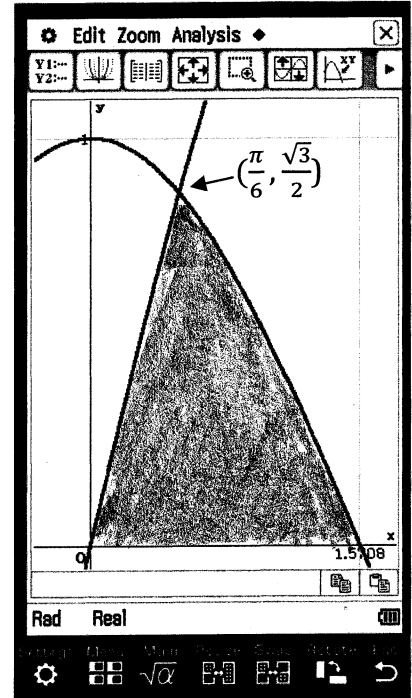Calculate the area between the graph of $f(x)$ and the $x - axis$.

Question 3 (5 marks)

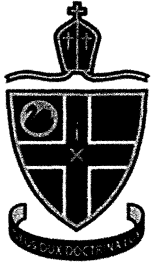The graphs of $f(x) = \cos(x)$ and $g(x) = \sqrt{3}\sin(x)$, in the first quadrant, are displayed to the right.

Find the shaded area bounded by the $x-axis, f(x)$ and $g(x)$.

Express your answer in its simplest form.

End of questions

Additional working space

Question number: _____

Christ Church Grammar School

MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name _____

Teacher's name _____

Time and marks available for this section
Reading time before commencing work: 3 minutes
Working time for this section: 30 minutes
Marks available: 25 marks

Materials required/recommended for this section
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Formula Sheet (retained from Section One)

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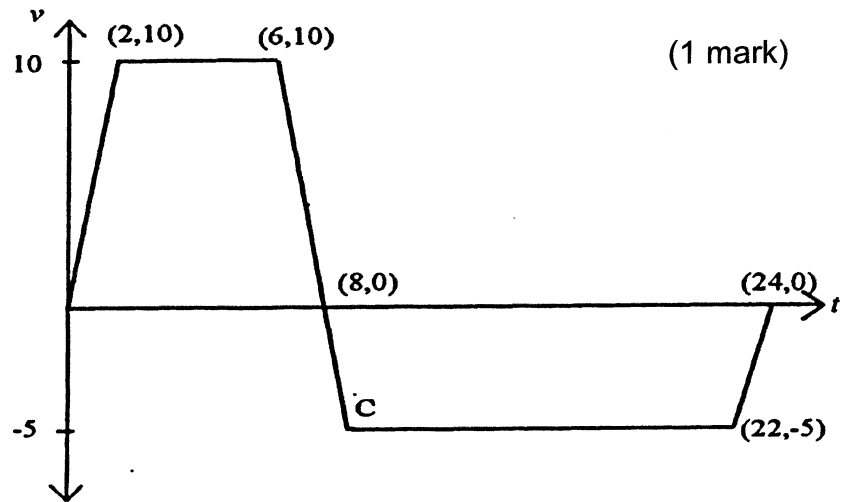7.   It is recommended that **you do not use pencil**, except in diagrams.

See next page

Question 4   **(7 marks)**

The graph below shows the velocity v (m/s) of a particle graphed against t (seconds) initially at the origin.

(a)   State the co-ordinate of C.   **(1 mark)**

(b)   State the acceleration at $t = 3$.   **(1 mark)**

(c)   Determine the acceleration during $0 \leq t \leq 2$.   **(1 mark)**

(d)   Determine the distance travelled in the first 10 seconds.   **(2 marks)**

(e)   Determine the time when the particle returns to its starting point?   **(2 marks)**

Question 5   **(5 marks)**

Consider the functions $f(x) = \frac{1}{2}(x^2 - 5x)\sqrt{x}$ and $g(x) = -3\sqrt{x}$. Points O $(0,0)$, A and B are the points of intersection of the graphs of the functions f and g.

(a)   Determine the coordinates of the points A and B, correct to 2 decimal places.

         (2 marks)

(b)   Use a definite integral to write an expression for the area enclosed by the graphs of the functions f and g and evaluate this integral, correct to 2 decimal places.

         (3 marks)

**Question 6**   **(8 marks)**

A particle moves along the $x - axis$ so that its acceleration $a(t)$ at any time t is given by $a(t) = 6t - 12m/s^2$. At time $t = 0$ the particle is instantaneously at rest at the point $x = 1$.

(a)   Write formulae for the velocity $v(t)$ and the displacement $x(t)$ of the particle, t seconds from the origin.   (2 marks)

(b)   Determine when and where the particle is again instantaneously at rest. (2 marks)

(c)   Determine the speed of the particle at $t = 2$.   (2 marks)

(d)   Determine the distance travelled by the particle between the times $t = 0$ and $t = 8$.   (2 marks)

See next page

Question 7   **(5 marks)**

The graph below shows the functions $f(x) = 3x$ and $g(x) = \dfrac{x^2}{2}$ and the line $x = 8$.
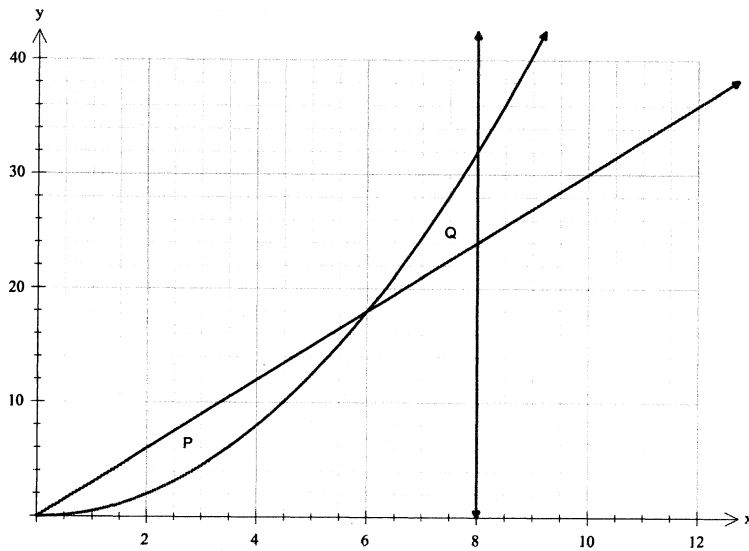
Region P is the area enclosed between f and g.

Region Q is the area enclosed by f, g and $x = 8$.

(a)   Determine the areas of P and Q.   (2 marks)

(b) $f(x)$ is re-defined such that $f(x) = ax$ and the area of Region P is half the area of Region Q. Calculate the value of a that makes this statement true.   (3 marks)

End of questions

Additional working space

Question number: _____

Additional working space

Question number: _____

MATHEMATICS METHODS Year 12

Section One:
Calculator-free

Your name _____ • Solutions • _____

Teacher's name_____ \

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Working time for this section: 15 minutes
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7. It is recommended that **you do not use pencil**, except in diagrams.

Question 1   **(6 marks)**

(a)   The function with rule $g(x)$ has derivative $g'(x) = \sin(2\pi x)$.
      Given that $g(1) = \frac{1}{\pi}$, find $g(x)$.   (3 marks)

$$g(x) = -\frac{1}{2\pi} \cos(2\pi x) + c \qquad \checkmark \quad (\text{Anti-diff})$$

$$g(1) = -\frac{1}{2\pi} \cos(2\pi) + c$$

$$\therefore \quad -\frac{1}{2\pi}(1) + c = \frac{1}{\pi}$$

$$c = \frac{3}{2\pi} \qquad \checkmark \quad (c\text{-value})$$

$$\therefore \quad \boxed{g(x) = -\frac{1}{2\pi}\cos(2\pi x) + \frac{3}{2\pi}} \quad \checkmark \quad \left(\begin{array}{c}g(x) \\ \text{with } `c\text{'}\end{array}\right)$$

(b)   If $\int_1^4 f(x)\,dx = 6$, then find the value of $\int_1^4 (5 - 2f(x))\,dx$.   (3 marks)

$$\int_1^4 5 - 2f(x)\,dx = \int_1^4 5\,dx - 2\int_1^4 f(x)\,dx$$

$$\checkmark \; (\text{re-write } \textstyle\int)$$

$$= \left[5x\right]_1^4 - 2 \times 6 \qquad \checkmark \quad (\text{Subs correctly})$$

$$= 20 - 5 - 12$$
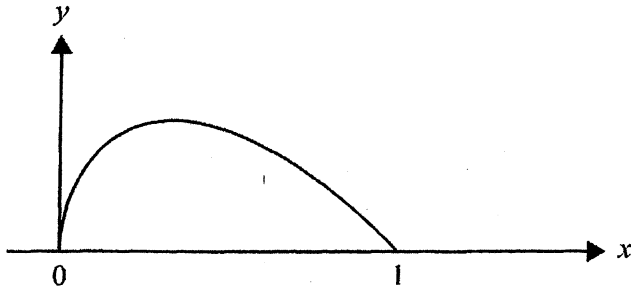
$$\boxed{= 3} \qquad \checkmark \quad (\text{Answ})$$

6

Question 2   **(4 marks)**

The graph of $f(x) = \sqrt{x}(1-x)$ for $0 \le x \le 1$ is shown below.

Calculate the area between the graph of $f(x)$ and the $x - axis$.

$$A = \int_0^1 \sqrt{x}(1-x)\,dx$$

$$A = \int_0^1 x^{1/2} - x^{3/2}\,dx \quad \checkmark \left(\begin{array}{c} re\text{-}write \\ integral \end{array} \right)$$

$$= \left[\frac{2}{3}x^{3/2} - \frac{2}{5}x^{5/2} \right]_0^1 \quad \checkmark \left(\int correctly \right)$$

$$= \frac{2}{3} - \frac{2}{5} \quad \checkmark \quad (subs\ in\ correctly)$$

$$= \frac{4}{15}\ units^2 \quad \checkmark \quad (Answer)$$
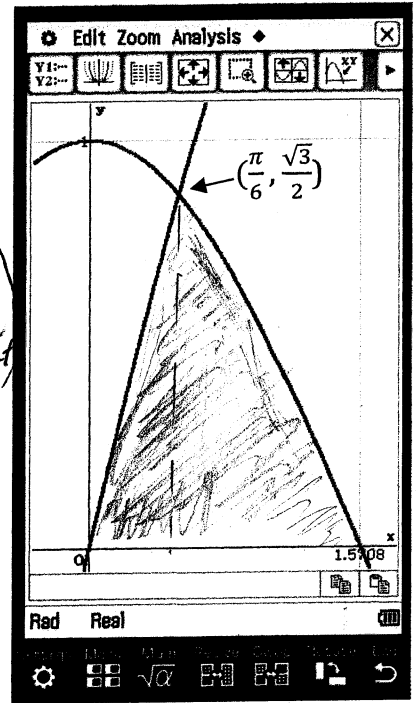
4

See next page

Question 3   **(5 marks)**

The graphs of $f(x) = \cos(x)$ and $g(x) = \sqrt{3}\sin(x)$, in the first quadrant, are displayed to the right.

Find the shaded area bounded by the $x - axis$, $f(x)$ and $g(x)$.

Express your answer in its simplest form.

$$\sqrt{3}\int_0^{\frac{\pi}{6}} \sin(x)\, dx + \int_{\frac{\pi}{6}}^{\frac{\pi}{2}} \cos(x)\, dx \qquad \checkmark \left(\begin{array}{c}\text{correct} \\ \text{integral} \\ \text{statement}\end{array}\right)$$

$$= \sqrt{3}\left[-\cos(x)\right]_0^{\frac{\pi}{6}} + \left[\sin(x)\right]_{\frac{\pi}{6}}^{\frac{\pi}{2}} \qquad \checkmark \left(\begin{array}{c}\text{Integrates} \\ \text{correctly}\end{array}\right)$$

$$= \sqrt{3}\left[-\cos\frac{\pi}{6} - (-\cos 0)\right] + \left[\underbrace{\sin\frac{\pi}{2}}_{1} - \underbrace{\sin\frac{\pi}{6}}_{\frac{1}{2}}\right]$$

$$= \sqrt{3}\left(-\frac{\sqrt{3}}{2} + 1\right) + \left(\frac{1}{2}\right) \qquad \checkmark\checkmark \left(\begin{array}{c}\text{uses exact values} \\ \text{in each bracket}\end{array}\right)$$

$$= -\frac{3}{2} + \sqrt{3} + \frac{1}{2}$$

$$= \underline{\sqrt{3} - 1 \quad \text{units}^2} \qquad \checkmark \left(\text{simplest form}\right)$$

. End of questions .

5
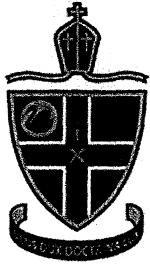
MATHEMATICS METHODS Year 12
Section Two:
Calculator-assumed

Your name _____ • SOLUTIONS • _____

Teacher's name _____

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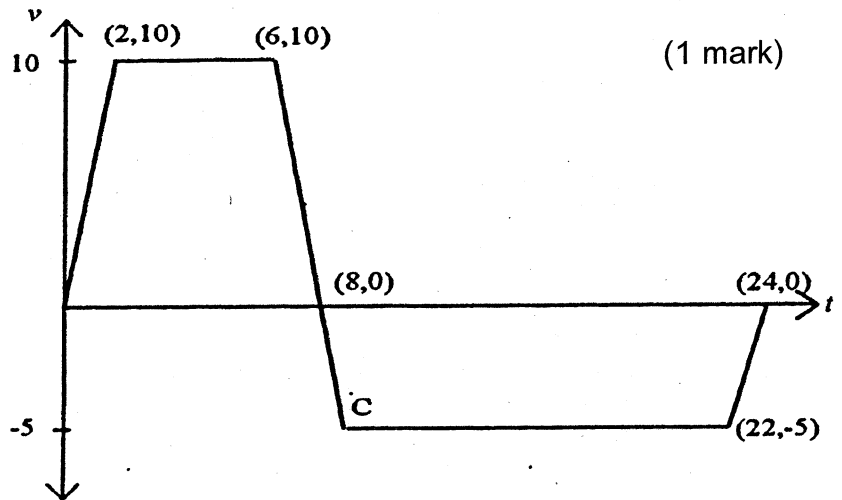7. It is recommended that **you do not use pencil**, except in diagrams.

Question 4 **(7 marks)**

The graph below shows the velocity v (m/s) of a particle graphed against t (seconds) initially at the origin.

(a) State the co-ordinate of C. (1 mark)

$m = -5$

$C\left(9, -5\right)$ ✓
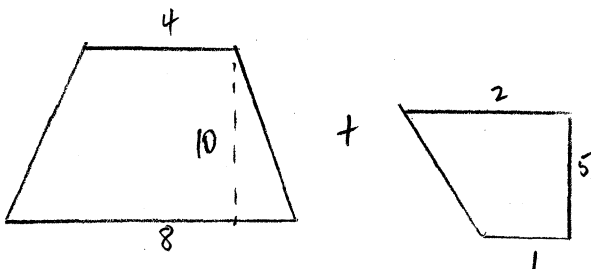

(b) State the acceleration at $t = 3$. (1 mark)

$a = \underline{0 \, m/s^2}$ ✓

(c) Determine the acceleration during $0 \le t \le 2$. (1 mark)

$a(t) = \underline{5 \, m/s^2}$ ✓

(d) Determine the distance travelled in the first 10 seconds. (2 marks)
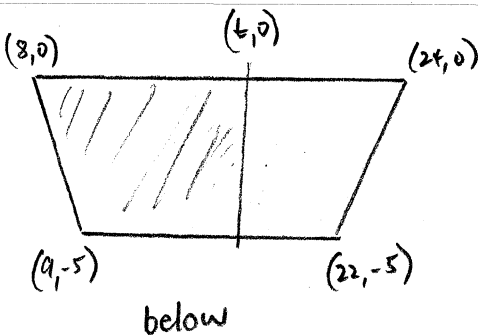
$A = \frac{1}{2}(4+8) \times 10 = 60$ ✓ both parts

$+ \; A = \frac{1}{2}(2+1) \times 5 = 7\frac{1}{2}$

$\underline{\underline{67\frac{1}{2} \, m}}$ ✓ total

(e) Determine the time when the particle returns to its starting point? (2 marks)

Area Above = Area below. ✓ (equate areas)

$60 = \frac{1}{2}\big[(t-8) + (t-9)\big] \times 5$

$60 = \frac{1}{2}(2t - 17) \times 5$

$\underline{\underline{t = 20.5 \, sec}}$ ✓ (time)

below

See next page

Question 5   **(5 marks)**

Consider the functions $f(x) = \frac{1}{2}(x^2 - 5x)\sqrt{x}$ and $g(x) = -3\sqrt{x}$. Points O $(0,0)$, A and B are the points of intersection of the graphs of the functions f and g.

(a)   Determine the coordinates of the points A and B, correct to 2 decimal places.

                                                                          (2 marks)

Class Pad $A(2, -4.24)$ ✓

                   $B(3, -5.20)$ ✓

(b)   Use a definite integral to write an expression for the area enclosed by the graphs of the functions f and g and evaluate this integral, correct to 2 decimal places.

                                                                          (3 marks)

$$\int_0^2 f(x) - g(x) \, dx + \int_2^3 g(x) - f(x) \, dx$$

(1st part) ✓ (2nd part) ✓ $\underline{OR}$ $\int_0^3 \left| g(x) - f(x) \right| dx$

$= 1.616244 \quad + \quad 0.131629$

$= \underline{1.75 \; v^2} \quad (\text{to } 2dp)$ ✓

$$\begin{bmatrix} -1 & \text{overall if} \\ & \text{not to } 2dp \end{bmatrix}$$

Question 6   **(8 marks)**

A particle moves along the $x - axis$ so that its acceleration $a(t)$ at any time t is given by $a(t) = 6t - 12 m/s^2$. At time $t = 0$ the particle is instantaneously at rest at the point $x = 1$.

(a)   Write formulae for the velocity $v(t)$ and the displacement $x(t)$ of the particle, t seconds from the origin.   (2 marks)

$$V(t) = \underline{3t^2 - 12t} \qquad As \; c = 0. \qquad \checkmark v(t)$$

$$x(t) = \underline{t^3 - 6t^2 + 1} \qquad As \; c = 1 \qquad \checkmark a(t)$$

(b)   Determine when and where the particle is again instantaneously at rest.   (2 marks)

$$when \quad v(t) = 0$$

$$3t(t-4) = 0$$

$$\boxed{t = 4} \; at \; rest \qquad \checkmark \; (time)$$

$$x(4) = \boxed{-31 m} \qquad \checkmark \quad (place)$$

(c)   Determine the speed of the particle at $t = 2$.   (2 marks)

$$|v(2)| = 3(2)^2 - 12(2)$$

$$= 12 - 24$$

$$= |-12| \qquad \checkmark \; velocity$$

$$= \boxed{12 \; m/s} \qquad \checkmark \; speed$$

(d)   Determine the distance travelled by the particle between the times $t = 0$ and $t = 8$.   (2 marks)
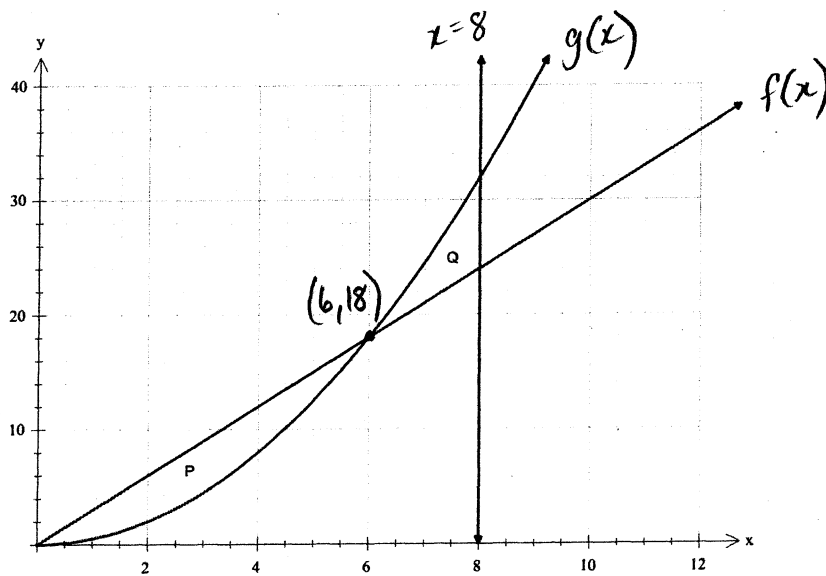
$$\int_0^8 |3t^2 - 12t| \; dt = \boxed{192 m}$$

$$\checkmark \; integral \qquad\qquad \checkmark Answ$$

See next page

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Question 7   **(5 marks)**

The graph below shows the functions $f(x) = 3x$ and $g(x) = \dfrac{x^2}{2}$ and the line $x = 8$.

Region P is the area enclosed between f and g.

Region Q is the area enclosed by f, g and $x = 8$.

(a)   Determine the areas of P and Q.   (2 marks)

Area $P = 18 u^2$ ✓

Area $Q = 7\frac{1}{3} u^2$ ✓

(b) $f(x)$ is re-defined such that $f(x) = ax$ and the area of Region P is half the area of Region Q. Calculate the value of a that makes this statement true.   (3 marks)

$ax = \dfrac{x^2}{2}$ (Intersection $g(x) \neq f(x)$) $\Rightarrow x = 2a$ ✓ (x-value)

Need 'a' such that

$$2 \int_0^{2a} ax - \frac{x^2}{2}\, dx = \int_{2a}^{8} \frac{x^2}{2} - ax\, dx \quad ✓ \text{ (Equates areas)}$$

ClassPad $a = \underline{2.384}$ ✓   (a-value)

End of questions

5